

Effective field theory and cold Fermi gases near unitary limit.

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Ultracold Fermi-gases near the unitary limit are studied in the framework of Effective Field Theory. It is shown that, while one can obtain a reasonable description of the universal proportionality constants both in the narrow and the broad Feshbach resonance limits, the requirement of the reparametrisation invariance leads to appearance of the three body forces needed to cancel the otherwise arising off-shell uncertainties. The size of the uncertainties is estimated.

Ultracold Fermi gases have recently attracted a lot of attention (see [1] and references therein) due to exiting possibility of tuning the strength of the fermion-fermion interaction through the Feshbach resonances so that the scattering length may become much larger then the typical scale in the system. This large scattering length is the main dynamical factor resulting in establishing the so called unitary limit (UL) which is believed to be universal in a sense that the only relevant energy scale is given by that of the non-interacting Fermi gas

$$E_{GS} = \xi E_{FG} = \xi \frac{3}{5} \frac{k_F^2}{2M}, \quad (1)$$

where M and k_F are the fermion mass and Fermi momentum correspondingly and ξ is the universal proportionality constant, which does not depend on the details of the interaction. The other dimensional characteristics of the cold Fermi-gas in the UL such as paring energy or chemical potential can also be represented in the same way.

Large scattering length implies nonperturbative treatment. The most “direct” nonperturbative method is based on the fixed-node MC approach [2, 3]. However, being potentially the most powerful calculational tool, direct numerical simulations still have many limitations related to finite size effects, discretization errors, trial wave function dependence etc which may even become amplified in certain physical situations (the system of several fermion species is one possible example). All that makes the development of the analytic approaches indispensable. Several such approaches have been suggested so far ranging from the exact renormalisation group [4, 5] and expansion in terms of dimensionality of space [6] to more

phenomenological approaches using the density functional method [7] and many body variational formalism [8]. The “world average” for the value of ξ is 0.42 ± 0.002 . It is important to emphasise that UL refers to the idealised situation with an infinite scattering length and vanishing effective radius which is the case of a broad Feshbach resonance. Even small but finite effective radius sets the other scale so that the system may deviate from the strict UL. This is the case in the narrow Feshbach resonance limit and also in nuclear/neutron matter where the experimental value of the effective radius is only one order of magnitude smaller than the scattering length and should therefore be taken into account. The purpose of this paper is to analyse the system of the cold Fermi atoms in the both narrow and broad Feshbach resonance limits in the framework of effective field theory (EFT) and study the general constraints EFT imposes on theoretical approaches describing the system of cold Fermi atoms in both UL and nearby.

EFT is based on the fact that the low scale dynamics is only weakly dependent on the details of the interaction at small distances. The low scale phenomena can then be described by a local effective lagrangian with some effective coupling constants reflecting the short range dynamics in some effective, indirect way. The physical amplitudes which can be derived from this lagrangian take the general form of the expansion in powers of the low scales involved with implicit assumption that all low scale are “natural” in a sense that all the possible dimensionless ratios are of order unity. As we have already mentioned the main feature of the cold Fermi atoms in UL is the large value of the scattering length. It makes the use of the canonical EFT impossible as the large scattering length introduces a new “unnatural” scale to the problem so that the power expansion is no longer valid. To overcome this difficulty it was suggested to iterate the leading term of the interaction to all orders by solving the Lippmann - Schwinger (LS) equation and to treat the rest as a perturbation [9, 10].

Being a proper field theory EFT must be regularised and renormalised. Besides, it must satisfy the reparametrisation invariance requirement which means that, although one can choose different representations for the field operators in the effective Lagrangian, the physical amplitudes should remain the same in any representation. In formal field theory this statement is known as a equivalence theorem [11]. In a more phenomenological language it means that the on-shell observables must be independent on the parametrisation used for the off-shell part of the fermion-fermion interaction. In the context of many fermion

systems with arbitrary scattering length the physical consequences of the reparametrisation invariance were considered in [12]. In this paper we use the findings of [12] to analyse the system of cold Fermi atoms at and around UL. The other important general requirement, usually called renormalisation group (RG) invariance, is the independence of the on-shell physics on the renormalisation parameters like cutoff or subtraction point. To comply with RG invariance the fermion-fermion scattering amplitude must satisfy the RG equation.

According to EFT the physical amplitudes at low scale can be derived from the effective Lagrangian with purely short range interactions. The corresponding LS equation for the fully off-shell amplitude $T(k', k, p)$ takes the form

$$T(k', k, p) = V(k', k, p) + M \int \frac{dq q^2}{2\pi^2} V(k', q, p) \frac{T(q, k, p)}{p^2 - q^2 + i\epsilon}. \quad (2)$$

Here we use k and k' to denote relative momenta and the energy dependence is given by $p = \sqrt{ME}$, the on-shell momentum corresponding to the centre-of-mass energy E . One possible form of the interaction can be written as

$$V = V_1 = C_0 + C_2' p^2. \quad (3)$$

Since it depends on on-shell momentum the interaction is purely energy dependent. The interaction can be written in a separable form so that the LS equation can easily be solved analytically. The resulting T -matrix takes the form

$$\frac{1}{T(p)} = \frac{(C_2 I_3 - 1)^2}{C_0 + C_2^2 I_5 + k^2 C_2 (2 - C_2 I_3)} - I(p), \quad (4)$$

where the loop integrals are

$$I_n \equiv -\frac{M}{(2\pi)^2} \int dq q^{n-1}, \quad (5)$$

and

$$I(p) \equiv \frac{M}{2\pi^2} \int dq \frac{q^2}{p^2 - q^2}. \quad (6)$$

These loop integrals are divergent and therefore the procedure of regularisation and renormalisation must be carried out. As a side remark we note that the issue of the nonperturbative renormalisation is quite a subtle problem. In contrast to the standard perturbative case where the usual field theoretical methods can be used to regularise the given divergent

graphs and then renormalize the bare coupling constants, in the nonperturbative situation the renormalisation of the whole integral equation must be carried out. In the case when the analytic solution for the scattering amplitude can be obtained, as is the case here, the renormalisation of the amplitude is a rather straightforward procedure. However, if the explicit solution is not possible (if the long range forces are added, for example) then the special care is needed to perform the renormalisation in a consistent way. In this paper we follow the procedure used in Ref. [13] to renormalize the effective fermion-fermion amplitude in vacuum. We subtract the divergent integrals at some kinematical point $p^2 = -\mu^2$ so that all the couplings should now depend on the subtraction point μ to ensure that the scattering amplitude is μ independent. The regularised fermion-fermion amplitude has the form

$$T_1 = \frac{C_0 + p^2 C'_2}{1 + \frac{M}{4\pi}(C_0 + p^2 C'_2)(ip + \mu)}. \quad (7)$$

The same expressions can be obtained in the regularisation scheme considered in [10].

The coupling constants can be determined from the suitable observables. For example, in the renormalisation scheme adopted in [10] they can be related to the low energy observables as

$$C_0(\mu) = \frac{4\pi}{M} \left(\frac{1}{-\mu + 1/a} \right), \quad C'_2(\mu) = \frac{M}{4\pi} C_0^2(\mu) \frac{r}{2}, \quad (8)$$

where a and r are the scattering length and the effective radius correspondingly. Strictly speaking the expression for the V_1 is written up to next-to-leading order term in the small scale according to the counting scheme developed in [10]. The LO coupling C_0 scales as p^{-1} and should therefore be treated nonperturbatively. The rest can be interpreted as a perturbation. However, it is rather easy to solve the whole Lippmann - Schwinger equation and obtain the vacuum T -matrix. It is worth mentioning that in the simplest case of the pointlike interactions all EFT does is just complicated way of getting the well known phenomenological effective range expansion. The full strength of the EFT can be easily realised when considering more complicated situations like, for example, the interaction with external currents. The straightforward generalisation of the phenomenological approaches may lead to the conflict with gauge invariance. The EFT solves this problem in a natural way treating all the interaction terms on an equal footing. That's the main motivation of using more general approach even in the case when it can be reduced to the well known phenomenological approaches.

It is clear that the above written purely energy dependent form of interaction is not unique. There may exist more general form of interaction in which the energy and momentum can be treated as formally independent variables. It has the form

$$V = V_2 = C_0 + C'_2 p^2 + \frac{1}{2} C_2 (\mathbf{k}^2 + \mathbf{k}'^2 - 2p^2), \quad (9)$$

where \mathbf{k} and \mathbf{k}' denote the initial and final relative momenta of the fermions and the coupling C_2 describes a purely off-shell interaction. The solution of the Lippmann - Schwinger equation is again straightforward and we get

$$T_2 = T_1 \left[1 + \frac{1}{2(C_0 + p^2 C'_2)} \left(C_2 (\mathbf{k}^2 + \mathbf{k}'^2 - 2p^2) - \frac{M}{8\pi} C_2^2 (p^2 - \mathbf{k}^2)(p^2 - \mathbf{k}'^2)(ip + \mu) \right) \right], \quad (10)$$

where T_1 is given by Eq. (7).

One can see from this equation that both amplitudes coincide on-shell so that the reparametrisation invariance is fulfilled.

The situation becomes much more complicated in the presence of medium. The corresponding amplitude can be calculated by solving the Feynmann-Galitskii equation

$$T^m = V + V G^F G^F T^m, \quad (11)$$

where G^F is the in-medium one-fermion propagator

$$G^F(\tilde{k}) = \frac{\theta(k - p_F)}{k_0 - \omega_k + i\epsilon} + \frac{\theta(p_F - k)}{k_0 - \omega_k - i\epsilon}, \quad (12)$$

here $\tilde{k} \equiv (k_0, \mathbf{k})$ and $\omega_k \equiv k^2/2M$.

For the interaction V_1 the solution of this equation takes the form

$$T_1^m = \left[\frac{1}{C_0(\mu) + p^2 C'_2(\mu)} + \frac{M\mu}{4\pi} - \frac{M(p_F + Q/2)}{4\pi^2} + \frac{M}{4\pi^2} \left(p \log \frac{p_F + Q/2 + p}{p_F + Q/2 - p} + \frac{Q^2/4 - p_F^2 + p^2}{\pi Q} \log \frac{(p_F + Q/2)^2 + p^2}{p_F^2 - Q^2/4 - p^2} \right) \right]^{-1}, \quad (13)$$

where p_F is the Fermi momentum and the amplitude is written for the case of the non-zero total momentum Q of the interacting fermion pair. Having determined the in-medium fermion-fermion amplitude we can extract the universal constant ξ by computing the energy density E_{GS} of the interacting Fermi gas and using the Eq.(1). The expression for the E_{GS} is given by

$$E_{GS} = E_{FG} + E_{int}, \quad (14)$$

where

$$E_{int} = \frac{3}{2\pi^2 p_F^5} \left[\int_0^{2p_F} Q^2 dQ \int_0^{p_F - Q/2} p^2 dp T^m(p^2, Q) + \int_0^{2p_F} Q dQ \int_{p_F - Q/2}^{\sqrt{p_F^2 - Q^2/4}} p dp T^m(p^2, Q) (p_F^2 - p^2 - Q^2/4) \right] \quad (15)$$

Calculation of the universal constant ξ in the genuine UL does not involve energy dependent part of the fermion-fermion interaction so we can drop the second term in Eq.(3). We obtain $\xi(UL) \simeq 0.33$ in the unitary limit with vanishing effective radius. Similar results were obtained in [14] using the effective range expansion for the fermion-fermion scattering amplitude in vacuum. In the language of EFT it corresponds to a particular choice of the subtraction point $\mu = 0$ so for the lowest-order effective coupling we get $C_0(\mu = 0) = 4\pi a/M$. We see that the effective coupling becomes arbitrary large in the UL so that it is hard to extract the effective couplings from observables and to justify the EFT expansion for the effective lagrangian in this case. As shown in [10] each individual graph in the sum of the bubble diagrams for the fermion-fermion scattering amplitude goes as $(4\pi a/M)(iap)^L$, where L is the number of loops so that each contribution in the bubble sum is bigger then the preceding one. It means that there is no well defined expansion parameter and it may result in artificial dependence on short range physics, unacceptable situation for consistent EFT. We emphasise, however that there is nothing wrong in using the phenomenological expressions like effective range expansion for the fermion-fermion amplitude to calculate the values for some physical quantities like $\xi(UL)$. In fact, a proper use of EFT leads precisely to this. Therefore, there is no wonder that EFT and phenomenological description lead to the similar results in this (simplest) case. One only needs to keeps in mind the constraints and limitations of the phenomenological approach. One side remark is that EFT provides a natural way of incorporating gauged interactions and external currents in unambiguous was, the opportunity often missing in phenomenological approaches where the conflict with gauge invariance is rather rule then exception. It is clear that EFT and phenomenological approaches are very different on this level of complexity. Therefore, it looks more advantageous to use EFT even in the simple cases where a suitable analytic parametrization of the fermion-fermion amplitude like effective range expansion can formally provide similar answers.

It is important to emphasise that the particle-particle (and hole-hole) summation with

undressed propagators represented by the FG equation provides only the simplest many-body approximation to the many-body problem. The further complications come from the self-energy and vertex corrections involving particle-hole pairs. However, they should start contributing at the order, prescribed by the power counting. It seems possible that for the dilute systems the counting should not be much different from that suggested in [10] for the fermion-fermion amplitude in vacuum since each loop involving the at least one hole line leads to the contribution proportional to p_F which is small so that the leading contribution is given by the summed particle-particle ladder in accord with the power counting from [10]. It is then looks conceivable that self-energy and vertex corrections will contribute at higher order. There is however a subtlety here. Let's consider the Eq. (13) for the elementary fermion-fermion amplitude and take for simplicity the case with zero total momentum. The corresponding expression for the T matrix can be written as

$$T_1^m = \frac{1}{\frac{1}{T_1} + \frac{M}{4\pi^2} [p \log \frac{p+p_F}{p-p_F} - 2p_F]}, \quad (16)$$

In the strict UL case the term with log does not contribute and the amplitude scale as $1/p_F$. Being combined with the loop it leads to the contribution of order one. It means that particle-particle and hole-hole ladders as well as particle-hole rings should in principle be treated on the same footing. Formulation of the power counting prescription is an open problem in this case and the use of the either simplest approximation like FG equation or more complicated approaches with self-energy and vertex corrections equally requires a further justification. If the in-medium power counting issue is resolved then all the improvements and corrections can be taken into account in a systematic way.

The situation seems to be somewhat different for the system with the finite effective range ($p_F r \sim 1$) and nonnegligible shape parameter (see below). In such a system these effects may lead to some additional suppression for the loop diagrams involving holes so that in this case counting indeed is closer to the vacuum one and the corresponding self-energy and vertex corrections are of higher order and thus suppressed. In this case the use of the FG equation seems more justified. Of course, the relative contributions of the suppressed and unsuppressed terms depend on a concrete value assumed for the effective radius. It is important however to keep the values of the effective range and shape parameter within the limits of applicability of the effective range expansion. We stress that these arguments are

rather crude and a consistent power counting is yet to be formulated. For example, it is not quite clear what kind of diagrams should contribute at the next-to-leading order and at what value of $p_F r$ should the counting start deviating from the vacuum one. All these issues need to be clarified in the future EFT-based studies of strongly interacting Fermi systems (dilute atomic Fermi gases, neutron/nuclear matter etc). We also note that the available experimental data on cold fermionic atoms correspond to the case of the broad Feshbach resonance with $p_F r \ll 1$ so that in this context the system of cold fermionic atoms with a large scattering length and the finite and modest effective radius refers to some future experiments. On the other hand the many-fermion system of this type is realised in nuclear/neutron matter.

With the effective range effects included we have obtained the value $\xi = 0.48$ at $p_F r \sim 1$. The result is rather close to the “world average” and seems to bring a certain credibility to the approach developed here. The results of calculations in the case of narrow Feshbach resonance with finite effective radius are shown on Fig.1 as a function of the dimensionless parameter $p_F r$. One notes, that the contribution due to the pairing interactions is known to be fairly small [15].

As one can see from the Fig.1 the corrections are quite significant already at $p_F r_e \sim 1$ and grow as the value of $p_F r$ increases. We found $\xi \simeq 0.68$ in the case of neutron matter with scattering length and effective radius being -18.5 fm and 2.7 fm correspondingly. The large value of the effective radius contribution suggests that one might expect the nonnegligible contribution from the next term of the low energy expansion of the effective fermion-fermion interaction which is proportional to $C_4 p^4$. The corresponding T-matrix in a free space takes the form

$$\frac{1}{T_1} = \frac{1}{C_0} + \frac{M}{4\pi}\mu - \frac{C'_2}{C_0^2}p^2 + \frac{C'_2}{C_0^2}\left(\frac{C'_2}{C_0} - \frac{C_4}{C_2}\right)p^4, \quad (17)$$

where we have written the T-matrix in a form, consistent with the counting rules suggested in [10]. The coupling C_4 entering the effective lagrangian at next-to-next leading order can be related to the so called shape parameter [10]. Again using the neutron matter parameters we found approximately $\xi_{C_4} \simeq 0.73$. We note that the size of the correction, while being non-negligible, suggests that the contributions of the higher order terms can be neglected.

All that looks reasonable but the word of caution is in order here. Let us turn to the more general case of the interaction V_2 with both energy and momentum dependence. The Feynmann-Galitskii equation can be solved in the same way and after putting the amplitude

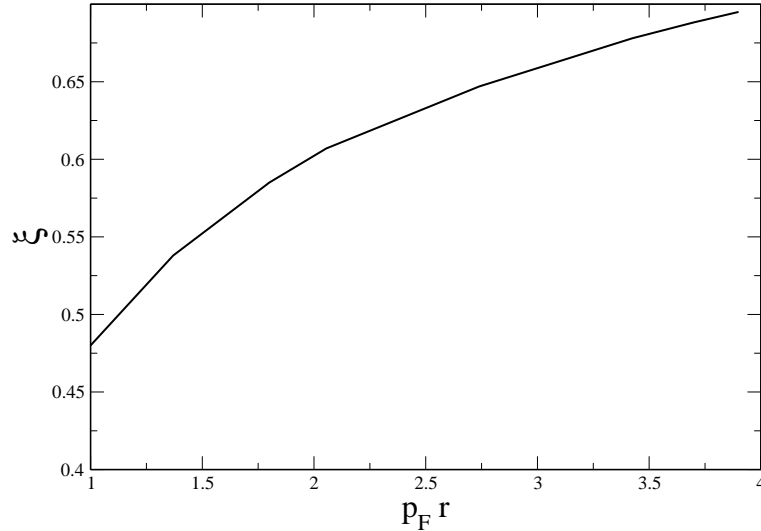


FIG. 1: The universal parameter ξ as the function of $p_F r$.

T_2^m on-shell we obtain

$$T_2^m = T_1^m - 2(T_1^m)^2 \frac{C_2(\mu)}{C_0(\mu)} \frac{M}{6\pi^2} p_F^3. \quad (18)$$

We see that two interactions which resulted in the same physical amplitudes in vacuum, lead to the different on-shell T -matrices in the presence of the fermion medium so that the reparametrisation invariance is not satisfied. Neither is satisfied the RG invariance requirement as the renormalisation performed at different subtraction points leads to different results for the physical observables. In other words, the physical observables still depend on the off-shell behaviour assumed for the fermion-fermion interaction. This is clearly the unsatisfactory situation which should be corrected. The general analysis of this problem was given in [12] so that here we give just a summary of the main points from [12]. Firstly, the hint on how to cancel the unphysical contributions comes from the second term in Eq.(17) which is proportional to the density. The same structure arises from a three-body (3B) contact interaction so that the 3B forces could probably be used to achieve the required cancellation. Secondly, we note that the contribution of the off-shell term is driven by the coupling constant C_2 which cannot be extracted from any physical observables which are defined on-shell. In the lowest order in the off-shell coupling C_2 the ground state energy is determined by the Hugenholtz diagrams shown in Fig. 2, where the solid dot denotes an in medium NN vertex and thick lines are dressed fermion propagators. Hugenholtz diagrams are versions of Feynman diagrams which explicitly incorporate antisymmetry of the inter-

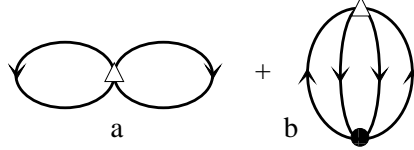


FIG. 2: Hugenholtz diagrams for the ground state energy at first order order in C_2 (the open triangle).

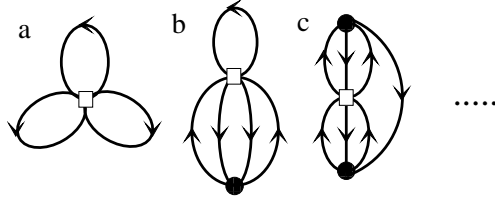


FIG. 3: Hugenholtz diagrams for the ground state energy at first order in the three-body force (the open square).

actions. Internal lines represent Feynman propagators which describe both particles and holes. The arrows represent the flow of quantum numbers such as baryon number. Each topologically distinct diagram should be multiplied by a symmetry factor to take account of the number of ways it can be constructed from the antisymmetric vertices. More details of these diagrams and the rules for evaluating them can be found in the textbooks [16, 17].

The diagrams in Fig. 2 give rise to many different contributions, which can be identified by iterating the equations for the in-medium NN vertex and dressed propagator. The approximations commonly used in many-body physics, typically amount to replacing the full NN vertex by a G - or T -matrix including both particle-particle ($p-p$) and hole-hole ($h-h$) ladders. Including ph rings as well as pp and hh ladders leads to the parquet approximation [16].

As shown in [12] the terms containing the unwanted off-shell contributions can indeed be exactly canceled against the contributions of a contact 3B interactions with three distinct topological structures shown in Fig.3. In the simplest Brueckner-Hartree-Fock approximation [16, 17], in which propagators are dressed and the in-medium NN vertex is obtained by iterating the potential in the $p-p$ and $h-h$ channels the unphysical contributions can be shown to cancel with Fig.3(a). When $p-h$ channel is added so that one iterates the interaction in all ($p-p$, $h-h$ and $p-h$) channels all three graphs from Fig.(3) are needed

to achieve the required cancellations.

We note that it is rather hard to make a rigorous statement on whether it is at all possible to formulate the general approach based on the 2B forces without the 3B ones and simultaneously satisfying the reparametrisation invariance requirement. The form of the second term in Eq.(17) seems to indicate that this is not possible but this is admittedly suggestive argument. In the context of the above discussion the more rigorous statement would be that for the most popular and widely used many-body approaches such as p - p/h - h ladder, parquet and even advanced parquet [16] approximations inclusion of the 3B forces is required to satisfy the reparametrisation invariance theorem. An additional support for this statement comes from the EFT studies of the few-body systems [18] where the 3B forces are needed to carry out a consistent renormalisation procedure. Moreover, as shown in [18] the corresponding 3B vertex must be promoted to a leading order in effective Lagrangian. All that strongly suggests that the 3B forces must necessarily be included at any level of truncations used so far in theoretical calculations.

The importance of the higher order diagrams with both the 2B and 3B forces could be estimated more quantitatively from some power counting rules. Unfortunately, as we pointed out above, establishing such a counting for the strongly interacting Fermi system is still an open and very challenging problem.

As we already mentioned, the off-shell parameters cannot be extracted from the on-shell physics so one can only theoretically estimate the off-shell contribution and, hence, the strength of the 3B forces needed to cancel it. The possible estimate could be based on the assumption that the term with the coupling C_2 gives the contributions of the same order as those related to its on-shell “cousin” C'_2 . It leads to

$$C_2 \sim C'_2 \sim 0(1), \quad (19)$$

if $\mu \sim p_F$. One notes that the 3B forces will also depend on the subtraction point μ to satisfy the RG invariance. Of course, the estimates obtained for the 3B forces are very crude and much quantitative work remains to be done to properly take their effect into account. Apart from the “destructive” role of cancelling the off-shell contributions the 3B forces should also play a “constructive” role in bringing the theory to better agreement with the data. The examples include the binding energy of the triton or low energy neutron-deuteron scattering [19]. The other example, which is very general and hold for any three body system with

infinite scattering length in the two body subsystems is the Efimov effect [20] which states that in such a system there exist infinitely many three-body bound states. It clearly should have a huge influence on the dynamics of the cold Fermi gases near the unitary limit and should therefore be taken into account in any consistent theoretical approach. Work in this direction is in progress.

The bottom line of this discussion is that, in spite of the fact that the EFT motivated studies of the Fermi gases near the unitary limit may result in the reasonable numbers for the physical observables they should be interpreted with a caution as neither power counting nor reparametrisation (and renormalisation) invariance issue is satisfactory implemented in theoretical schemes at present.

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